Lab 7

Standing Waves and Chladni Figures

A. Purpose

To study the resonance phenomena of the mechanical waves on a string, along a spring, and on metal of different shapes.

B. Introduction

Waves in nature have three catalogs: mechanical waves, field waves (electromagnetic waves and gravitational waves), and matter waves. It is relatively easy to observe the various phenomena of mechanical waves, yet not so easy to spot the other two. This experiment focuses on the resonances of mechanical waves and the patterns of standing waves.

Standing waves were first noticed by Michael Faraday in 1831 on the surface of a liquid in a vibrating container. In the 19th century, Franz Melde found the resonance patterns of a string between two fixed points and coined the term "standing wave" for this phenomenon. He also generated parametric oscillations in a string by employing a tuning fork to periodically vary the tension at twice the resonance frequency of the string.[1] Here, we follow Melde's idea to study standing waves on a string stretched between two fixed supports with one end subjected to a periodic vertical force, which causes traveling waves to propagate down the string with velocity

$$v = \sqrt{T/\mu} \tag{1}$$

where T is the tension in the string and μ is the mass per unit length of the string. When waves reflect off the far end of the string, they would interfere with incident waves, and for certain driving frequencies, a stable pattern emerges, characterized by fixed points of destructive interference (nodes) and constructive interference (anti-nodes). For this to occur,

$$L = n\lambda/2 \quad \text{with } n = 1, 2, 3, \ldots \text{ (integer)} \tag{3.2}$$

where λ is the wavelength of the standing waves as shown in Fig. 1, which tells that only certain wavelengths (frequencies) can exist as stable patterns, known as the "normal modes" of the system.

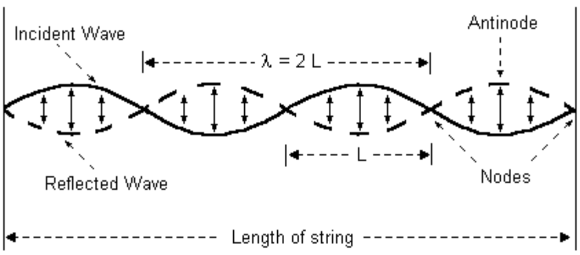

Fig. 1: Standing waves of a string between two fixed supports.

[1] Wikipedia contributors. (2020, February 25). Melde's experiment. In Wikipedia, The Free Encyclopedia. Retrieved 05:07, August 31, 2021, from https://en.wikipedia.org/w/index.php?title=Melde%27s_experiment&oldid=942581797

We can also observe features of standing waves such as nodes and antinodes in a spring that is fixed at one end and vibrated at the other end. A longitudinal wave sent down the length of a spring can be seen as a zone of compression followed by a zone of rarefaction. The wave will eventually reflect off the other end of the spring and travel back. If a second wave travels out as the first wave travels back, they will interfere with one another. Therefore, a standing wave (or resonance) appears in the spring as shown in Fig. 2 with nodes standing still and antinodes oscillating vigorously.

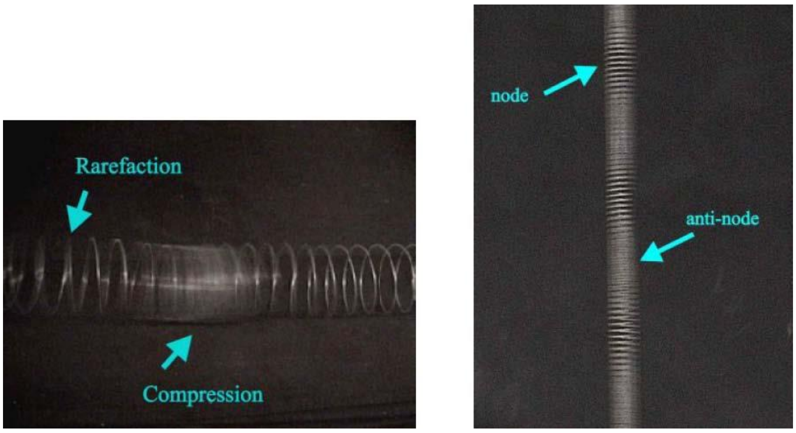

Fig. 2: Longitudinal standing wave in a spring

Standing waves can also be demonstrated in a two-dimensional plane. In the late 18th century, Ernst Chladni sprinkled fine sand evenly on a plate, oscillating the plate at a certain frequency. It was found that fine sand would stay on nodal lines, where there was no oscillation. Sand not on the nodal lines would bounce up and down with motions of oscillation until it reached nodal lines and stayed there. If the oscillating plate had uniform density, resonance patterns would change with the oscillating frequency, and different patterns, called Chladni figures, would emerge.

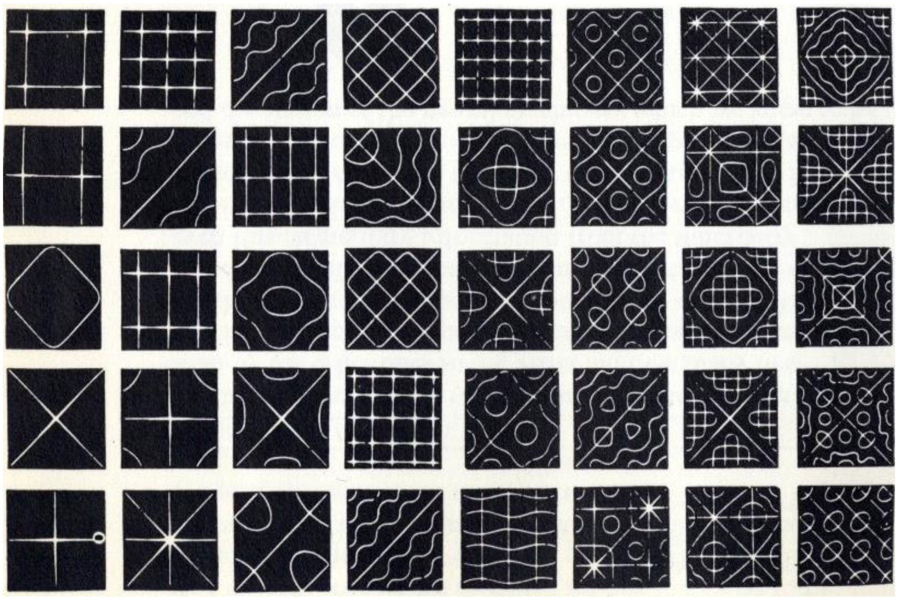

Fig. 4: Chladni patterns on a square plate published by John Tyndall in 1869.

Features of standing waves occur in modern physics as well. In 1913, Bohr formulated the well-known planetary model of the atom. In this model, an electron orbiting farther from the nucleus is in a higher energy state than one in a closer orbit, which cannot, however, well explain the fact that electrons occupy only certain energy levels. In 1924, Louis de Broglie presented the idea of matter waves, suggesting the electron be thought of as though its mass and charge spread out into a standing wave surrounding the atomic nucleus, which explains the certain energy levels of electrons orbiting the nucleus. In this experiment, a wire loop analogously demonstrates standing waves at discrete frequencies corresponding to unique wavelengths.

Fig. 3: Standing wave in a wire loop

C. Apparatus

PASCO SF-9324 Mechanical Wave Driver

PASCO WA9867 Sine Wave Generator

Spring

Metal Strips

Wire Loop

24 x 24 cm[2] Aluminum Plate, Sprinkle Bottle and Fine Sand

Violin-shaped Aluminum Plate

D. Procedures

1. Pre-lab assignments (hand in before the lab)

   Make a flowchart of this experiment and answer the questions

   (1) Read "Speed of Wave Pulses in Hooke's Law Media" by Elisha Huggins[2], and explain the difference between the speed v_C of a compression pulse on a spring and the speed v_T of a transverse wave in a spring. Moreover, why can the formula for v_C be independent of unstretched length L_0? (Explain in your own words without the tedious mathematics.)

   (2) Consider the experiment setup as shown in Fig. 5. The resonance pattern appears when the mechanical driver is driven by a function generator with amplitude of about 1 mm and some specific frequencies f_n. Suppose the total length of the hanging spring is L, and its mass is M with the spring constant k. Find f_n in terms of n, k, and M. Note that both ends of the spring are fixed. (Hint: What's the speed of the wave propagating in the spring?)

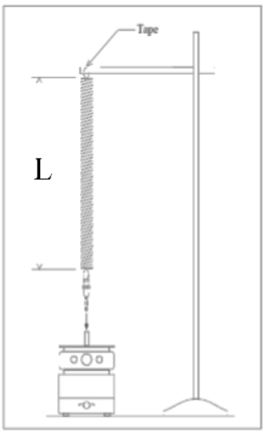

Fig. 5: Longitudinal wave along a spring

[2] Elisha Huggins , "Speed of Wave Pulses in Hooke's Law Media", The Physics Teacher 46, 142-146 (2008) https://doi.org/10.1119/1.2840977

(3) Explain why Chladni figures on symmetric plates are symmetric. Do not prove by mathematics; instead, give an interpretation by a physical picture.

2. In-lab activities

(1) Resonance of Waves on a one-dimensional string

(i) Measurement of the linear density μ $(\mathrm{kg/m})$.

Take a piece of string. Measure the physical quantities you need to obtain the linear density μ of the string, and calculate the results. Five independent measurements are needed for each quantity.

*** State your result in the standard form: $\mu = \mu_{\mathrm{best}} \pm \delta\mu$.

(ii) Determine the linear density μ_{ex} by transverse standing waves

(a) Set the mechanical wave driver, the wave generator, the pulley and the support rods as shown in Fig. 6. **"LOCK"** the wave driver during the setting. Note that you should attach the end of the string to Support rod as shown.

(b) Hang a mass on the pulley end of the string. Five trials are needed, and the choices of hanging mass are $m = 50,\ 60,\ 70,\ 80,\ 90\ \mathrm{g}$, by which you can calculate the tension T of the string for each trial.

(c) **"UNLOCK"** the wave driver and switch on the wave generator with sine signal. After adjusting the amplitude to proper value, increase the frequency slowly.

(d) Find the **three** resonant frequencies $f_n\,(\text{recommended } n: 2 \sim 4)$ for each trial. Measure the corresponding wavelengths λ_n, and calculate the speed v_n. State the speed $v_{\mathrm{m}} = v_{\mathrm{best,m}} \pm \delta v_{\mathrm{m}}$ for the m-th trial.

(e) Draw a graph of v^2 versus T with the results of all trials, and show the uncertainties with the help of error bars. Finally, find the linear density μ_{ex} by the graph, and compare the result with μ in the procedure (i).

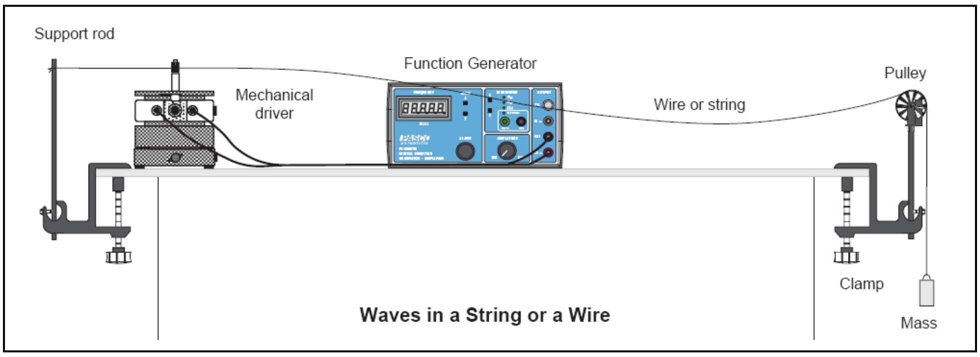

Fig. 6: Resonance of transverse waves on a one-dimensional string

(iii) **(Optional)** Parametric resonance (Melde's experiment)

(a) Rest the mechanical wave driver on the side with the rubber feet as Fig.7 shows and hang a $10\ \mathrm{g}$ mass on the pulley side.

(b) Find a resonant frequency, and the corresponding speed of waves. Describe and explain the differences between part (i) and part (ii) in your post-lab report. (Hint: Do you expect to excite transverse waves in this part?)

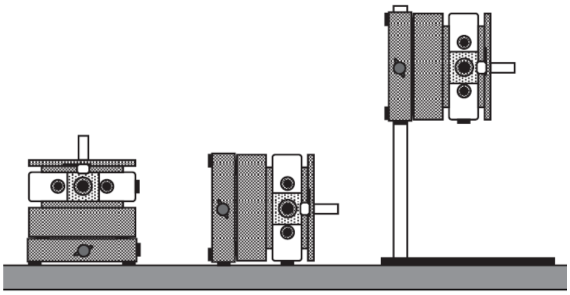

Fig. 7: Mount the mechanical wave driver on the side of the rod

(2) Resonance of Transverse Waves on metal strips (Cantilever)

(i) Set up the experiment as shown in Fig. 8. Rotate the metal strips so that they are at equal angles from each other, and measure the length of each strip.

(ii) **"LOCK"** the driver arm by sliding the drive arm locking tab to the Lock position. Insert the banana plug into the driver shaft of the PASCO Mechanical Driver. Then **"UNLOCK"** the drive arm.

(iii) Connect the mechanical driver to a function generator and start driving the mechanical driver at about 5 Hz with amplitude of about 1 mm and slowly increase the frequency. Find the resonant frequencies at which each of the strips vibrates with maximum amplitude. (When resonance is attained it may be necessary to decrease the driving amplitude so as to see a clear pattern.)

(iv) Suppose the relation between the resonant frequency f and the length L of the strip is $f \cong aL^n$ in this experiment. Find the value of n.

(v) **(Optional)** What if the metal strips are not at equal angles? Describe the difference you find in the post-lab report.

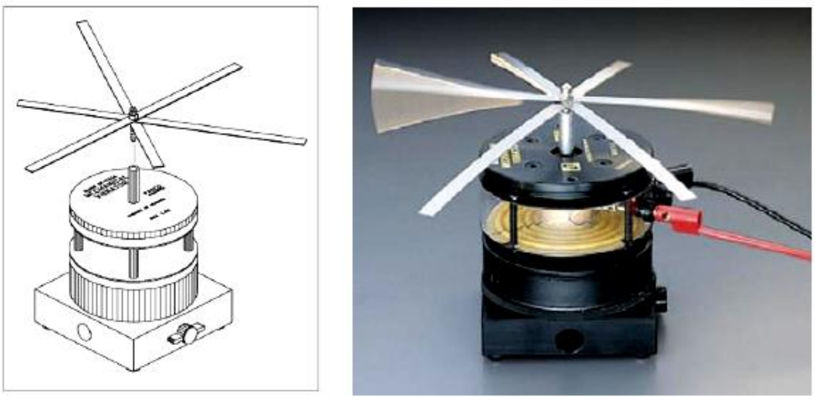

Fig. 8: Resonance of Transverse Waves on metal strips (Cantilever)

(3) Resonance of waves on a metal ring

Change the metal strips to the metal ring and follow the processes (ii) and (iii) in experiment 2. As the frequency increases, the wire loop will begin to vibrate in various modes. Find the relation between the resonant frequency f and its corresponding number m of nodes by exciting the modes of $m = 3, 5, 7, 9$. Use this relation to predict the frequency of the modes for $m = 4$ and $m = 11$. Find the experimental results and compare with the prediction.

(4) Resonance of longitudinal waves along a spring

To find the relation among the total length L of the spring, the number n of half wavelengths and the driving frequency f, excite the modes of $n = 7,8,9,10,11$ and change the total length L to re-do the experiment. Compare your result with the theory. (Hint: the question given in your pre-lab assignment)

(5) Resonance on two-dimensional plates (Chladni Figures)

Connect the Chladini plate to the drive shaft. Sprinkle sand on top of the plate. Unlock the drive shaft of the wave driver and vibrate the plate by changing the frequencies with a proper amplitude by the function generator. As the frequency increases, clear symmetric resonant patterns will emerge. Take at least **three** photos of the different resonant patterns for each plate.

3. Post-lab report
   (1) Recopy and organize your data from the in-lab tables in a neat and more readable form.
   (2) Analyze the data you obtained in the lab and answer the given questions
   (3) Compare the results with the theory, and discuss the uncertainties that occur in the experiments, and how they influence the experiments. (Quantitatively, if possible.)

E. Questions

1. In physics, when the speed of a wave depends on its frequency, the supporting medium is called "**dispersive**." In other words, in the material, the value of the group velocity $d\omega/dk$ is different from that of the phase velocity ω/k. From the results in experiments 1~4, explain whether they are dispersive or non-dispersive.
2. In experiment 3, the metal ring is used to demonstrate the electron standing wave pattern. However, in the Bohr model, the number of nodes observed would be an even number instead of an odd number as mostly seen in experiment 3. Describe the difference between this experiment and Bohr model, and explain why the number of nodes is odd in the experiment.
3. In experiment 4, you use the idea of Q2 in the pre-lab assignment to design an experiment of the resonance of longitudinal waves along a spring. Theoretically, we say this is a node-node standing wave (two fixed ends). However, one observes that the lowest node of this node-node standing wave is visible slightly above the lower end of the spring. Explain this phenomenon by providing physical reasoning.

4. In experiment 5, you have observed that Chladni figures depend on the frequency as well as the shape. If you are instead given three plates of same shapes but different materials now, do you expect to see the same resonant patterns on them? Explain.

F. References

Wikipedia contributors. (2020, February 25). Melde's experiment. In Wikipedia, The Free Encyclopedia. Retrieved 05:07, August 31, 2021.

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